

# The *a priori* calculation of NMR nuclear spin–spin coupling constants

Trygve Helgaker, University of Oslo, Norway

N. Handy, M. Jaszuński, P. Lantto, O. Lutnæs, M. Pecul, T. Ruden, K. Ruud, P. Salek,  
J. Sadlej, J. Vaara, M. Watson

## Overview

- the mechanisms responsible for nuclear spin–spin couplings
- the calculation of indirect nuclear spin–spin couplings
- comparison of theoretical methods
  - what  $N$ -electron models and basis sets are needed?
  - are the results in qualitative or quantitative agreement with experiment?
  - how does DFT perform relative to wave-function theory?
- applications to small and large molecules

## Nuclear spin–spin coupling

- With each nuclear spin  $\mathbf{I}_P$  in a molecule, there is an associated magnetic moment:

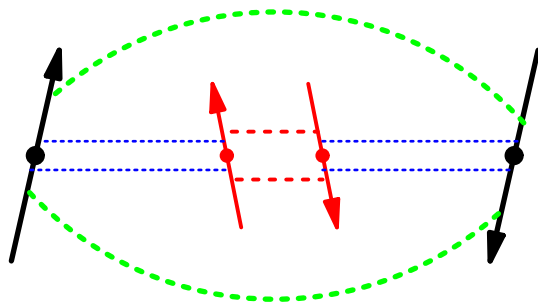
$$\mathbf{M}_P = \gamma_P \hbar \mathbf{I}_P \approx 10^{-4} \text{ a.u.}$$

- These moments interact **directly** with one another, by a classical dipole mechanism:

$$\mathbf{M}_P^T \mathbf{K}_{PQ}^{\text{direct}} \mathbf{M}_Q = \alpha^2 \frac{R_{PQ}^2 (\mathbf{M}_P \cdot \mathbf{M}_Q) - 3(\mathbf{M}_P \cdot \mathbf{R}_{PQ})(\mathbf{R}_{PQ} \cdot \mathbf{M}_Q)}{R_{PQ}^5} \approx 10^{-12} \text{ a.u.}$$

- the direct coupling is anisotropic and vanishes in isotropic media (gases and liquids)

- However, there is also an **indirect** contribution to the coupling, arising from **hyperfine interactions** with the intervening electrons:



- the indirect coupling is exceedingly small:

$$\approx 10^{-16} \text{ a.u.} \approx 1 \text{ Hz}$$

- it does not vanish in isotropic media
- fine structure of high-resolution NMR spectra

- Analogy with molecular electrostatic interactions may be helpful:

- the bare nuclei repel one another by a classical Coulomb mechanism
- this interaction is modified by the intervening electrons, which couple to the nuclei
- the combined interactions may provide stable molecular structures

## Indirect nuclear spin–spin coupling tensors

- To second order in the nuclear moments, the change in the **electronic energy** is described in terms of the (reduced) indirect spin–spin coupling tensors  $\mathbf{K}_{PQ}$ :

$$E(\mathbf{M}_P) = E(\mathbf{0}) + \sum_{P>Q} \mathbf{M}_P^T \mathbf{K}_{PQ} \mathbf{M}_Q + \mathcal{O}(\mathbf{M}^3)$$

- for closed-shell molecules, there are no first-order contributions
- the  $\mathbf{K}_{PQ}$  are simply the second derivatives of  $E(\mathbf{M}_P)$  at  $\mathbf{M}_P = \mathbf{0}$
- they may be calculated by standard techniques for second-order molecular properties
- In what manner is the electronic energy affected by the magnetic moments?
- The nuclear moments set up a magnetic **vector potential** and a **field** ( $\approx 10^{-8}$  a.u.):

$$\mathbf{A}(\mathbf{r}) = \alpha^2 \sum_P \frac{\mathbf{M}_P \times \mathbf{r}_P}{r_P^3}, \quad \mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

- In the presence of  $\mathbf{A}$  and  $\mathbf{B}$ , the **kinetic-energy operator** is modified and becomes:

$$H_{\text{kin}} = \frac{1}{2} p^2 + \underbrace{\mathbf{A}(\mathbf{r}) \cdot \mathbf{p}}_{\text{orbital hyperfine}} + \frac{1}{2} A(\mathbf{r})^2 + \underbrace{\mathbf{B}(\mathbf{r}) \cdot \mathbf{s}}_{\text{spin hyperfine}}$$

- In nonrelativistic theory, therefore, there are two distinct contributions to the coupling:
  - hyperfine coupling to the **orbital motion** of the electrons
  - hyperfine coupling to the **spin** of the electrons

## Orbital-hyperfine interactions

- The orbital hyperfine or spin-orbit (SO) coupling represents the interaction of charged particles (electrons) moving in the vector potential  $\mathbf{A}(\mathbf{r})$  generated by the nuclei.
- There are two types of orbital hyperfine operators:

$$\frac{1}{2}A^2 = \sum_{PQ} \mathbf{M}_P^T \mathbf{h}_{PQ}^{\text{DSO}} \mathbf{M}_Q, \quad \mathbf{h}_{PQ}^{\text{DSO}} = \alpha^4 \frac{\mathbf{r}_P^T \mathbf{r}_Q \mathbf{I}_3 - \mathbf{r}_P \mathbf{r}_Q^T}{r_P^3 r_Q^3} \leftarrow \text{diamagnetic SO (DSO)}$$

$$\mathbf{A} \cdot \mathbf{p} = \sum_P \mathbf{M}_P^T \mathbf{h}_P^{\text{PSO}}, \quad \mathbf{h}_P^{\text{PSO}} = \alpha^2 \frac{\mathbf{r}_P \times \mathbf{p}}{r_P^3} \leftarrow \text{paramagnetic SO (PSO)}$$

- Whereas the second-order real DSO operator contributes to the expectation value, the first-order imaginary PSO operator contributes to second order in perturbation theory:

$$\mathbf{K}_{PQ}^{\text{SO}} = \langle 0 | \mathbf{h}_{PQ}^{\text{DSO}} | 0 \rangle + 2 \sum_{n_S \neq 0} \frac{\langle 0 | \mathbf{h}_P^{\text{PSO}} | n_S \rangle \langle n_S | (\mathbf{h}_Q^{\text{PSO}})^T | 0 \rangle}{E_{n_S} - E_0}$$

- The DSO contribution is calculated by numerical quadrature.
- The PSO contribution is calculated using linear response theory, in much the same manner as we calculate for example static polarizabilities.
- The SO contributions (especially DSO) are usually but not invariably small.
- For large internuclear separations, the DSO and PSO contributions cancel.

## Spin-hyperfine interactions

- To treat the spin-hyperfine interactions  $\mathbf{B} \cdot \mathbf{s}$ , we need the nuclear magnetic field:

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) = \frac{8\pi\alpha^2}{3} \sum_P \delta(\mathbf{r}_P) \mathbf{M}_P + \alpha^2 \sum_P \frac{3\mathbf{r}_P(\mathbf{r}_P \cdot \mathbf{M}_P) - r_P^2 \mathbf{M}_P}{r_P^5}$$

- This magnetic field  $\mathbf{B}(\mathbf{r})$  then gives rise to two distinct **first-order triplet operators**:

$$\mathbf{B} \cdot \mathbf{s} = \sum_P \mathbf{M}_P^T (\mathbf{h}_P^{\text{FC}} + \mathbf{h}_P^{\text{SD}}), \quad \left\{ \begin{array}{ll} \mathbf{h}_P^{\text{FC}} = \frac{8\pi\alpha^2}{3} \delta(\mathbf{r}_P) \mathbf{s} & \text{Fermi contact (FC)} \\ \mathbf{h}_P^{\text{SD}} = \alpha^2 \frac{3\mathbf{r}_P \mathbf{r}_P^T - r_P^2 \mathbf{I}_3}{r_P^5} \mathbf{s} & \text{spin-dipole (SD)} \end{array} \right.$$

where FC represents interaction **at the nucleus** and SD interaction **at a distance**.

- The FC and SD operators both contribute to second order in perturbation theory:

$$\mathbf{K}_{PQ}^{\text{FC+SD}} = 2 \sum_{n_T} \frac{\langle 0 | \mathbf{h}_P^{\text{FC}} + \mathbf{h}_P^{\text{SD}} | n_T \rangle \langle n_T | (\mathbf{h}_Q^{\text{FC}})^T + (\mathbf{h}_Q^{\text{SD}})^T | 0 \rangle}{E_{n_T} - E_0}$$

– The  $\mathbf{K}_{PQ}^{\text{FC+SD}}$  are evaluated by linear response theory, just like the  $\mathbf{K}_{PQ}^{\text{PSO}}$ .

- Relative importance of the terms:
  - the isotropic FC/FC term often dominates short-range coupling constants
  - the FC/SD and SD/FC terms often dominate the anisotropic part of  $\mathbf{K}_{PQ}$

## 50 years of Ramsey's theory

- The expression for the indirect spin–spin coupling tensors  $\mathbf{K}_{PQ}$  was first derived by Ramsey in 1953:

$$\begin{aligned}
 \mathbf{K}_{PQ} &= \langle 0 | \mathbf{h}_{PQ}^{\text{DSO}} | 0 \rangle \quad \leftarrow \text{diamagnetic orbital (singlet)} \\
 &+ 2 \sum_{n_S \neq 0} \frac{\langle 0 | \mathbf{h}_P^{\text{PSO}} | n_S \rangle \langle n_S | (\mathbf{h}_Q^{\text{PSO}})^{\text{T}} | 0 \rangle}{E_{n_S} - E_0} \quad \leftarrow \text{paramag. orbital (singlet)} \\
 &+ 2 \sum_{n_T} \frac{\langle 0 | \mathbf{h}_P^{\text{FC}} + \mathbf{h}_P^{\text{SD}} | n_T \rangle \langle n_T | (\mathbf{h}_Q^{\text{FC}})^{\text{T}} + (\mathbf{h}_Q^{\text{SD}})^{\text{T}} | 0 \rangle}{E_{n_T} - E_0} \quad \leftarrow \text{spin (triplet)}
 \end{aligned}$$

- In 1950, Ramsey had already derived an expression for **nuclear shieldings**  $\sigma_P$ , describing the interaction of the nuclei with an external magnetic field in the presence of electrons.
- Together,  $\sigma_P$  and  $\mathbf{K}_{PQ}$  determine the **spin Hamiltonian** of high-resolution NMR:

$$H_{\text{NMR}} = -\mathbf{B}^{\text{T}} \sum_P (\mathbf{I}_3 - \sigma_P) \mathbf{M}_P + \sum_{P>Q} \mathbf{M}_P^{\text{T}} \mathbf{K}_{PQ} \mathbf{M}_Q$$

- experimentalists determine  $\sigma_P$  and  $\mathbf{K}_{PQ}$  from observed spectra
- our task is to determine these from the molecular electronic structure

## Spin–spin coupling constants by linear response theory

- In linear response theory, we may express the  $\alpha\beta$  Cartesian component of the spin–spin coupling tensor  $\mathbf{K}_{PQ}$  in this manner:

$$K_{P\alpha,Q\beta} = \left\langle h_{P\alpha,Q\beta}^{\text{DSO}} \right\rangle - [\mathbf{F}_{P\alpha}^{\text{PSO}}]^T [\mathbf{G}_{\text{ii}}^{\text{ss}}]^{-1} \mathbf{F}_{Q\beta}^{\text{PSO}} - \sum_{\gamma} [\mathbf{F}_{P\alpha\gamma}^{\text{FC+SD}}]^T [\mathbf{G}_{\text{rr}}^{\text{tt}}]^{-1} \mathbf{F}_{Q\gamma\beta}^{\text{FC+SD}}$$

- Right-hand side elements:

$$[\mathbf{F}_P^{\text{PSO}}]_{ai} = 2\alpha^2 \left\langle a \left| \frac{\mathbf{l}_P}{r_P^3} \right| i \right\rangle, \quad [\mathbf{F}_P^{\text{FC+SD}}]_{ai} = \alpha^2 \left\langle a \left| \frac{8\pi}{3} \delta(\mathbf{r}_P) \mathbf{I} + \frac{3\mathbf{r}_P \mathbf{r}_P^T - r_P^2 \mathbf{I}}{r_P^5} \right| i \right\rangle$$

- Leading Hessian terms are diagonal orbital-energy differences:

$$\begin{aligned} [\mathbf{G}_{\text{ii}}^{\text{ss}}]_{ai,bj} &= \delta_{ij} \delta_{ab} (\varepsilon_a - \varepsilon_i) + \gamma (g_{ajib} - g_{abij}) \quad \leftarrow \text{diagonal in pure DFT} \\ [\mathbf{G}_{\text{rr}}^{\text{tt}}]_{ai,bj} &= \delta_{ij} \delta_{ab} (\varepsilon_a - \varepsilon_i) - \gamma (g_{ajib} + g_{abij}) + \int \frac{\partial^2 f}{\partial \rho_t^2} \phi_a \phi_i \phi_b \phi_j \mathbf{dr} + \dots \end{aligned}$$

- In practice, Hessian inversion is avoided by the solution of linear equations:

$$\mathbf{G}_{\text{rr}}^{\text{tt}} \boldsymbol{\lambda}_{Q\gamma\beta}^{\text{FC+SD}} = -\mathbf{F}_{Q\gamma\beta}^{\text{FC+SD}}$$

- There are a total of 9 (10) such equations to be solved for each nucleus.

## Calculations of indirect spin–spin calculations

- The calculation of spin–spin coupling constants is **challenging task**:
  - triplet as well as singlet perturbations are involved
  - electron correlation important—the Hartree–Fock model fails abysmally
  - the dominant FC contribution requires an accurate description of the electron density at the nuclei (large decontracted  $s$  sets)
- We must solve a large number of response equations:
  - 3 singlet equations and 7 triplet equations for each nucleus
  - for shieldings, only 3 equations are required, for molecules of all sizes
- Spin–spin couplings are very sensitive to the molecular geometry:
  - equilibrium structures must be chosen carefully
  - large vibrational corrections (often 5%–10%)
- However, unlike in shielding calculations, there is no need for London orbitals since no external magnetic field is involved.
- For heavy elements, a relativistic treatment may be necessary:
  - four-component theory, ZORA (Pyykkö, Saue, Ziegler)

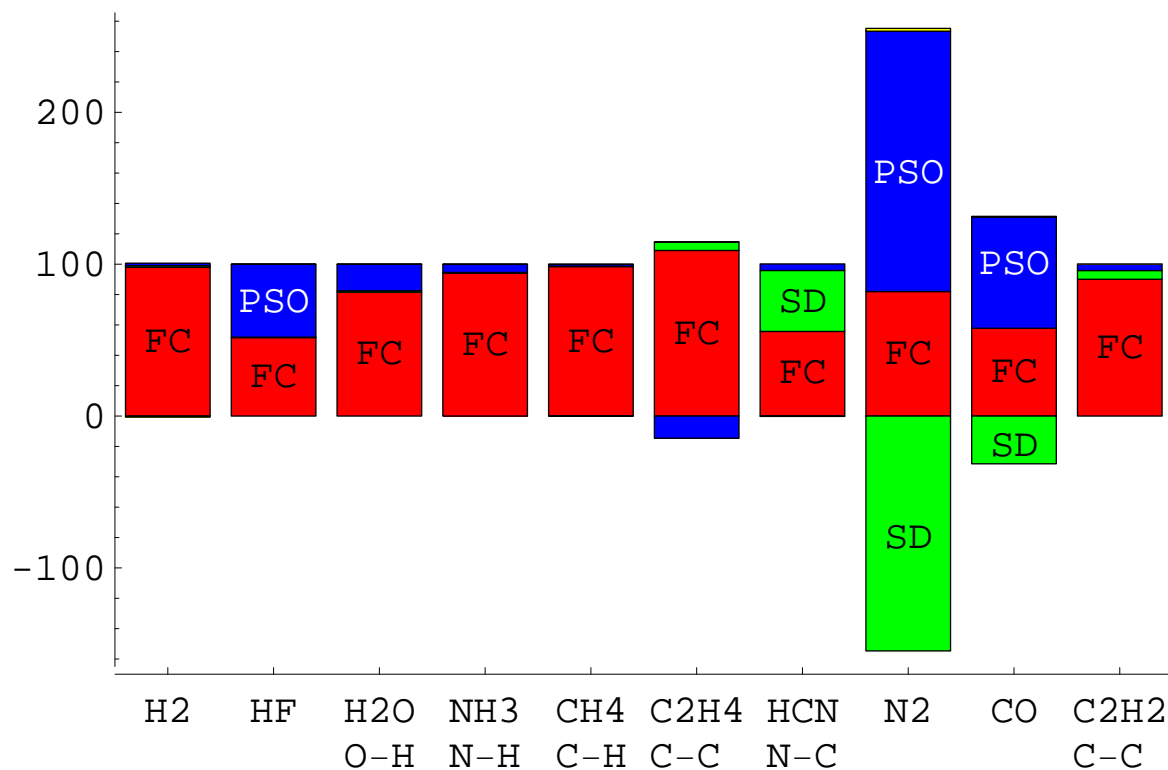


## Relative importance of the contributions to spin–spin coupling constants

- The isotropic indirect spin–spin coupling constants can be uniquely decomposed as:

$$\mathbf{K}_{PQ} = \mathbf{K}_{PQ}^{\text{DSO}} + \mathbf{K}_{PQ}^{\text{PSO}} + \mathbf{K}_{PQ}^{\text{FC}} + \mathbf{K}_{PQ}^{\text{SD}}$$

- The spin–spin coupling constants are often dominated by the FC term.
- Since the FC term is relatively easy to calculate, it is tempting to ignore the other terms.
- However, none of the contributions can be *a priori* neglected (N<sub>2</sub> and CO)!

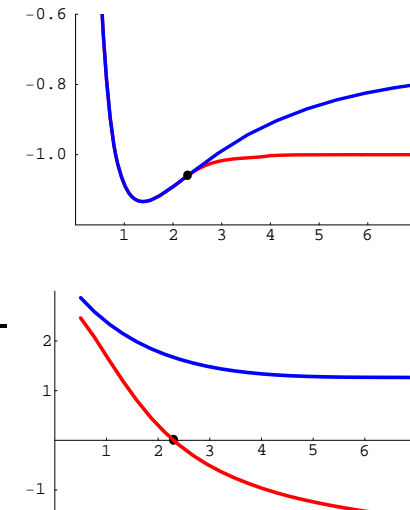


## RHF and the triplet instability problem

- RHF **does not** in general work for spin–spin calculations:

- the RHF wave function often becomes triplet unstable
- at or close to such instabilities, the RHF description of spin interactions becomes unphysical
- the spin–spin coupling constants of  $C_2H_4$ :

Hz	$^1J_{CC}$	$^1J_{CH}$	$^2J_{CH}$	$^2J_{HH}$	$^3J_{cis}$	$^3J_{trans}$
exp.	68	156	–2	2	12	19
RHF	1270	755	–572	–344	360	400
CAS	76	156	–1	3	14	21



- Indeed, any method **based on the RHF reference state** may have problems:

- $^1J_{CN}$  in HCN [Auer and Gauss, JCP 115 (2001) 1619]

Hz	RHF	CCSD	CCSD(T)	CC3	CCSDT
relaxed	–92.0	–8.1	7.4	–2.6	–14.5
unrelaxed		–15.0		–14.7	–14.6

- in CC theory, orbital relaxation should be treated through singles amplitudes
- noniterative CCSD(T) cannot be used; iterative CC3 must be used instead
- all electrons must be correlated in unrelaxed CC calculations

## Correlated methods for spin–spin calculations

- For the reliable calculation of spin–spin coupling constants, there is clearly a need for **correlated** methods.
- Traditional **wave-function methods** used for spin-spin calculations:
  - **CISD** (Kowalewski, Roos, Siegbahn and Vestin, 1974)
  - **SOPPA** (Geertsen and Oddershede, 1984)
  - **MCSCF** (Vahtras, Ågren, Jørgensen, Jensen, Padkjær and Helgaker, 1992)
    - \* static correlation with CAS
    - \* dynamical correlation with RAS
  - **EOM-CCSD** (Perera, Sekino and Bartlett, 1994)
  - **CCSD(T), CC3, CCSDT** (Auer and Gauss, 2001)
- Most widely used: SOPPA, MCSCF, EOM-CCSD
- Typical **errors** are a few Hz in small systems, but often 10%–20% for small couplings
- However, all of these methods are (at least for now) restricted to **small systems**.

Reduced spin–spin coupling constants ( $10^{19} \text{kg m}^{-2} \text{s}^{-2} \text{\AA}^{-2}$ )

		RHF	CAS	RAS	SOPPA	CCSD	CC3	exp*	vib
HF	$^1J_{\text{HF}}$	59.2	48.0	48.1	46.8	46.1	46.1	47.6	−3.4
CO	$^1J_{\text{CO}}$	13.4	−28.1	−39.3	−45.4	−38.3	−37.3	−38.3	−1.7
N <sub>2</sub>	$^1J_{\text{NN}}$	175.0	−5.7	−9.1	−23.9	−20.4	−20.4	−19.3	−1.1
H <sub>2</sub> O	$^1J_{\text{OH}}$	63.7	51.5	47.1	49.5	48.4	48.2	52.8	−3.3
	$^2J_{\text{HH}}$	−1.9	−0.8	−0.6	−0.7	−0.6	−0.6	−0.7	0.1
NH <sub>3</sub>	$^1J_{\text{NH}}$	61.4	48.7	50.2	51.0	48.1		50.8	−0.3
	$^2J_{\text{HH}}$	−1.9	−0.8	−0.9	−0.9	−1.0		−0.9	0.1
C <sub>2</sub> H <sub>4</sub>	$^1J_{\text{CC}}$	1672.0	99.6	90.5	92.5	92.3		87.8	1.2
	$^1J_{\text{CH}}$	249.7	51.5	50.2	52.0	50.7		50.0	1.7
	$^2J_{\text{CH}}$	−189.3	−1.9	−0.5	−1.0	−1.0		−0.4	−0.4
	$^2J_{\text{HH}}$	−28.7	−0.2	0.1	0.1	0.0		0.2	0.0
	$^3J_{\text{cis}}$	30.0	1.0	1.0	1.0	1.0		0.9	0.1
	$^3J_{\text{tns}}$	33.3	1.5	1.5	1.5	1.5		1.4	0.2
$ \bar{\Delta} $	abs.	180.3	3.3	1.6	1.8	1.2	1.6	*at $R_e$	
	%	5709	60	14	24	23	6		

## DFT calculations of spin–spin coupling constants

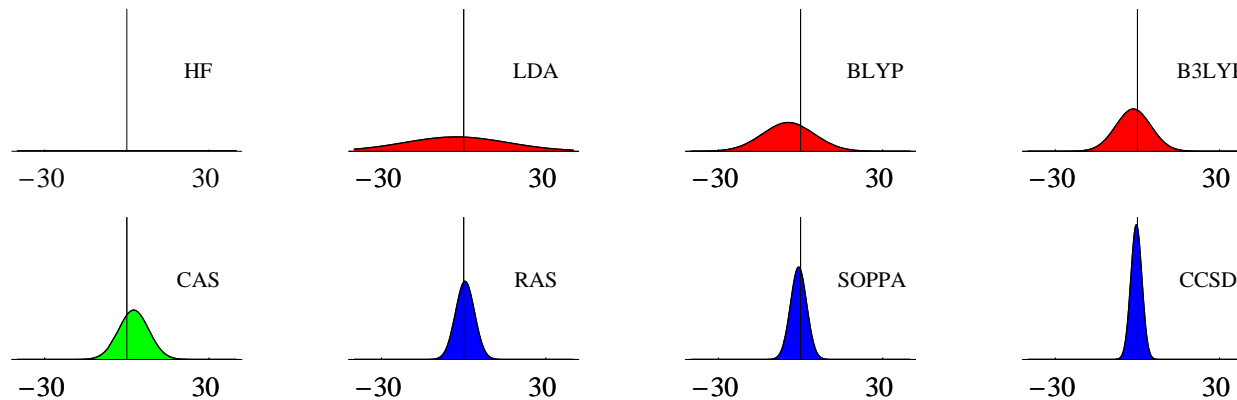
- DFT offers an alternative approach to spin–spin calculations:
  - Malkin, Malkina, Salahub (1994):  
LDA and GGA with neglect of SD contribution  
FPT & SOS; GTOs
  - Dickson and Ziegler (1996):  
LDA with neglect of SD contribution  
FPT & SOS; STOs
  - Sychrovský, Gräfenstein and Cremer; Helgaker, Watson and Handy (2000)  
LDA, GGA and hybrid DFT,  
response theory; GTOs
  - Autschbach and Ziegler (2000);  
LDA and GGA  
response theory; STOs; ZORA
- All of these apply standard exchange–correlation functionals
  - current density functionals should be tried!

Reduced spin–spin coupling constants ( $10^{19} \text{kg m}^{-2} \text{s}^{-2} \text{\AA}^{-2}$ )

		RHF	LDA	BLYP	B3LYP	RAS	exp*	vib
HF	$^1 J_{\text{HF}}$	59.2	35.0	34.5	36.8	48.1	47.6	−3.4
CO	$^1 J_{\text{CO}}$	13.4	−65.4	−55.7	−44.9	−39.3	−38.3	−1.7
N <sub>2</sub>	$^1 J_{\text{NN}}$	175.0	32.9	−46.6	−32.9	−9.1	−19.3	−1.1
H <sub>2</sub> O	$^1 J_{\text{OH}}$	63.7	40.3	44.6	46.6	47.1	52.8	−3.3
	$^2 J_{\text{HH}}$	−1.9	−0.3	−0.9	−0.6	−0.6	−0.7	0.1
NH <sub>3</sub>	$^1 J_{\text{NH}}$	61.4	41.0	49.6	52.6	50.2	50.8	−0.3
	$^2 J_{\text{HH}}$	−1.9	−0.4	−0.7	−0.8	−0.9	−0.9	0.1
C <sub>2</sub> H <sub>4</sub>	$^1 J_{\text{CC}}$	1672.0	66.6	90.3	98.3	90.5	87.8	1.2
	$^1 J_{\text{CH}}$	249.7	42.5	55.3	54.7	50.2	50.0	1.7
	$^2 J_{\text{CH}}$	−189.3	0.4	0.0	−0.4	−0.5	−0.4	−0.4
	$^2 J_{\text{HH}}$	−28.7	0.4	0.4	0.2	0.1	0.2	0.0
	$^3 J_{\text{cis}}$	30.0	0.8	1.1	1.1	1.0	0.9	0.1
	$^3 J_{\text{tns}}$	33.3	1.2	1.7	1.7	1.5	1.4	0.2
$ \bar{\Delta} $	abs.	180.3	11.2	5.9	4.2	1.6	*at $R_e$	
	%	5709	72	48	20	14		

## Comparison of density-functional and wave-function theory

- normal distributions of errors for these molecules and some other systems for which vibrational corrections have been made:



- some observations:
  - HF has a very broad distribution and overestimates strongly
  - LDA underestimates only slightly, but has a large standard deviation
  - BLYP reduces the LDA errors by a factor of two
  - B3LYP improves upon GGA (but not as dramatically as for other properties)
  - B3LYP errors are similar to those of CASSCF and about twice those of the dynamically correlated methods RASSCF, SOPPA, and CCSD
  - the most accurate method appears to be CCSD
  - the situation is much less satisfactory than for geometries and atomization energies

## DFT failures and successes

- DFT failures: couplings to electronegative and lone-pair atoms
  - fluorine couplings, in particular, are strongly underestimated:

		RHF	LDA	BLYP	B3LYP	CCSD	exp
HF	$^1J_{\text{HF}}$	669	396	390	417	522	538
FHF <sup>-</sup>	$^2J_{\text{FF}}$	657	-175	-113	25	439	≈ 274

- this is obviously a failure of the functionals rather than of DFT as such
- the KT1 functional of Keal and Tozer gives the following one-bond couplings:

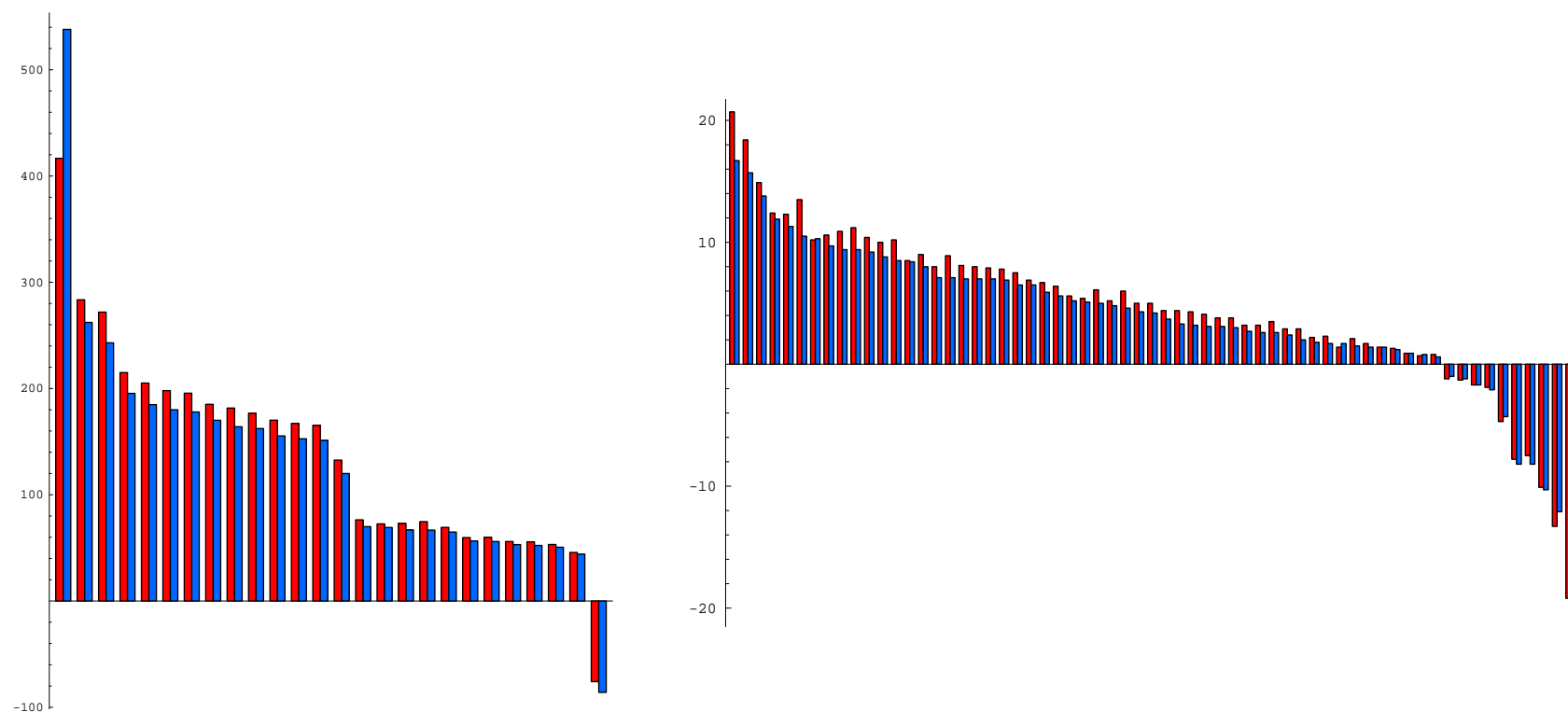
	HF	CO	NN	OH	NH	CC	CH	CC
KT1	507	19	1	-81	44	51	147	13
exp.	538	16	2	-86	44	67	151	35
B3LYP	417	18	3	-76	46	75	165	33

- Successes of DFT:
  - trends are well reproduced
  - hydrocarbons are well reproduced (Malkin, Malkina, and Salahub)  
mean abs. err. 4 Hz and std. dev. 6 Hz (vs. 5 and 15 Hz in general)  
even better with small basis sets :)



## Trends in spin–spin coupling constants

- comparison of calculated (red) and B3LYP (blue) spin–spin coupling constants
  - plotted in order of decreasing experimental value



- trends are quite well reproduced by B3LYP, in particular for large couplings

## Spin-spin coupling constants of hydrocarbons

	B3LYP	emp		B3LYP	emp		B3LYP	emp
<b>Benzene</b>			${}^2J_{C_2H_1}$	5.0	4.3	${}^3J_{C_2H_4}$	7.5	6.5
${}^3J_{HH}$	8.9	7.1	${}^3J_{C_2H_5}$	6.9	6.5	${}^1J_{C_2C_3}$	76.4	70.0
${}^4J_{HH}$	1.3	1.2	${}^3J_{C_3H_1}$	7.9	7.0	<b>Thiophene</b>		
${}^5J_{HH}$	0.8	0.6	${}^3J_{C_3H_5}$	8.0	7.0	${}^3J_{H_2H_3}$	6.0	4.6
${}^1J_{CH}$	167.1	152.7	${}^3J_{C_2H_4}$	8.1	7.0	${}^3J_{H_3H_4}$	4.4	3.3
${}^2J_{CH}$	2.1	1.5	${}^1J_{C_2C_3}$	73.0	66.9	${}^4J_{H_2H_5}$	3.2	2.6
${}^3J_{CH}$	8.0	7.1	${}^1J_{NH_1}$	72.6	69.2	${}^4J_{H_2H_4}$	0.9	0.9
${}^4J_{CH}$	-1.2	-1.0	${}^2J_{NH_2}$	3.8	3.1	${}^1J_{C_2H_2}$	198.0	180.0
${}^1J_{CC}$	59.9	56.0	${}^3J_{NH_3}$	4.4	3.7	${}^1J_{C_3H_3}$	176.7	162.3
${}^2J_{CC}$	-1.7	-1.7	${}^1J_{NC_2}$	10.2	10.3	${}^2J_{C_2H_3}$	9.0	8.0
${}^3J_{CC}$	11.2	9.4	${}^2J_{NC_3}$	3.8	3.0	${}^2J_{C_3H_2}$	5.4	5.1
<b>Pyrrole</b>			<b>Furan</b>			${}^2J_{C_3H_4}$	6.7	5.9
${}^3J_{H_1H_2}$	3.2	2.7	${}^3J_{H_2H_3}$	2.3	1.7	${}^3J_{C_2H_5}$	5.6	5.2
${}^3J_{H_2H_3}$	3.5	2.6	${}^3J_{H_3H_4}$	4.1	3.1	${}^3J_{C_3H_5}$	10.4	9.2
${}^3J_{H_3H_4}$	4.3	3.2	${}^4J_{H_2H_5}$	1.7	1.4	${}^3J_{C_2H_4}$	10.9	9.4
${}^4J_{H_2H_5}$	2.2	1.8	${}^4J_{H_2H_4}$	0.7	0.8	${}^1J_{C_2C_3}$	69.3	64.9
${}^4J_{H_1H_3}$	2.9	2.4	${}^1J_{C_2H_2}$	215.0	195.4	<b>Cyclopropane</b>		
${}^4J_{H_2H_4}$	1.4	1.4	${}^1J_{C_3H_3}$	185.1	170.1	${}^2J_{H,H}$	-4.7	-4.2
${}^1J_{C_2H_2}$	195.6	177.9	${}^2J_{C_2H_3}$	12.3	11.3	${}^3J_{HH}^{cis}$	10.3	8.6
${}^1J_{C_3H_3}$	181.6	164.0	${}^2J_{C_3H_2}$	14.9	13.8	${}^3J_{HH}^{trans}$	6.1	5.1
${}^2J_{C_2H_3}$	10.0	8.8	${}^2J_{C_3H_4}$	5.0	4.2	${}^1J_{CH}$	170.3	156.0
${}^2J_{C_3H_2}$	8.5	8.4	${}^3J_{C_2H_5}$	7.8	6.9	${}^2J_{CH}$	-1.9	-2.1
${}^2J_{C_3H_4}$	5.2	4.8	${}^3J_{C_3H_5}$	6.4	5.6	${}^1J_{CC}$	12.4	11.9

### Basis-set requirements

- Accurate spin–spin calculations require large basis sets, augmented with **steep  $s$  functions**
  - Huz-III- $su0$ : [11s6p2d/6s2p]
  - Huz-III- $su3$ : [14s6p2d/9s2p]
- B3LYP calculations on benzene with and without added  $s$  functions:

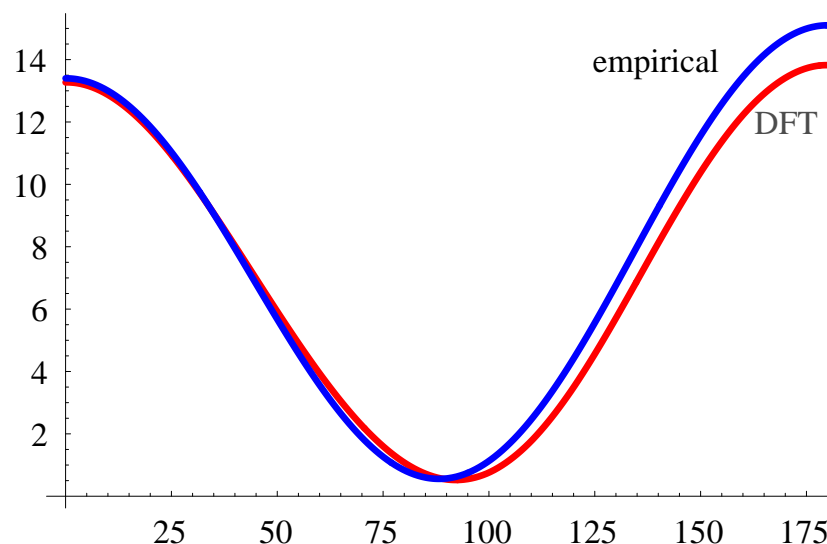
Hz	MCSCF	B3LYP- $su3$	B3LYP- $su0$	emp
$^1 J_{CC}$	70.9	60.0	56.7	56.1
$^2 J_{CC}$	-5.0	-1.8	-1.7	-1.7
$^3 J_{CC}$	19.1	11.2	10.7	9.4
$^1 J_{CH}$	176.7	166.3	151.7	152.7
$^2 J_{CH}$	-7.4	2.0	1.7	1.4
$^3 J_{CH}$	11.7	8.0	7.3	7.0
$^4 J_{CH}$	-1.3	-1.2	-1.3	-1.0
$^3 J_{HH}$		8.7	7.6	7.0
$^4 J_{HH}$		1.3	1.1	1.2
$^5 J_{HH}$		0.8	0.7	0.6

- **Cancellation of errors** gives excellent agreement without added steep functions :)

### The Karplus curve

- Vicinal couplings depend critically on the dihedral angle:

- $^3J_{\text{HH}}$  in ethane as a function of the dihedral angle:
- The agreement with the empirical Karplus curve is good.



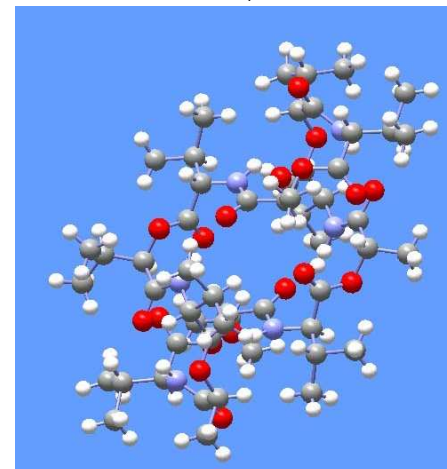
### NMR spin–spin coupling constants in $\text{C}_{60}$

- For the spin–spin coupling constants in  $\text{C}_{60}$ , we find (at the B3LYP level):
  - $^1J$  couplings within one pentagon and between two pentagons are 62 and 77 Hz.
  - $^2J$  couplings within one pentagon and between two pentagons are 7 and 1 Hz.
  - $^3J$  couplings are 4 Hz.
  - $^4J$  couplings and all other couplings are smaller than 1 Hz.

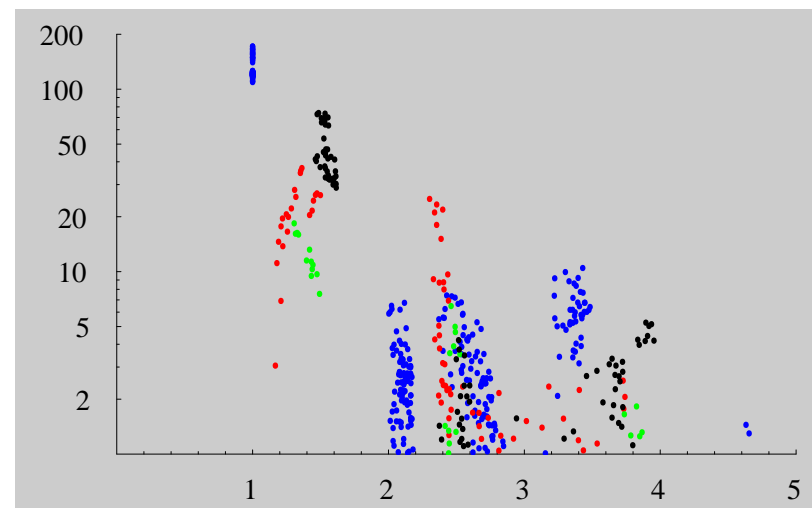
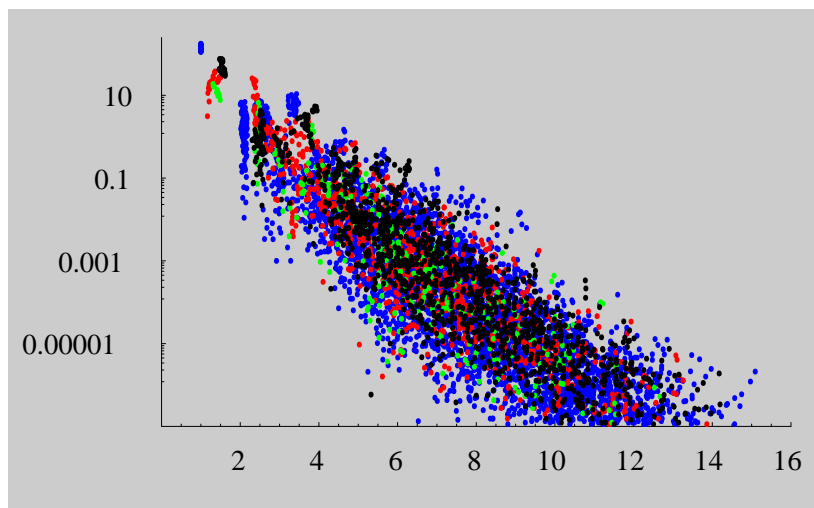
## Valinomycin $C_{54}H_{90}N_8O_{18}$

- DFT can be applied to large molecular systems such as valinomycin (168 atoms)

- There are a total of 7587 spin–spin couplings to carbon atoms in valinomycin.
- Most of the indirect couplings are small and cannot be detected.

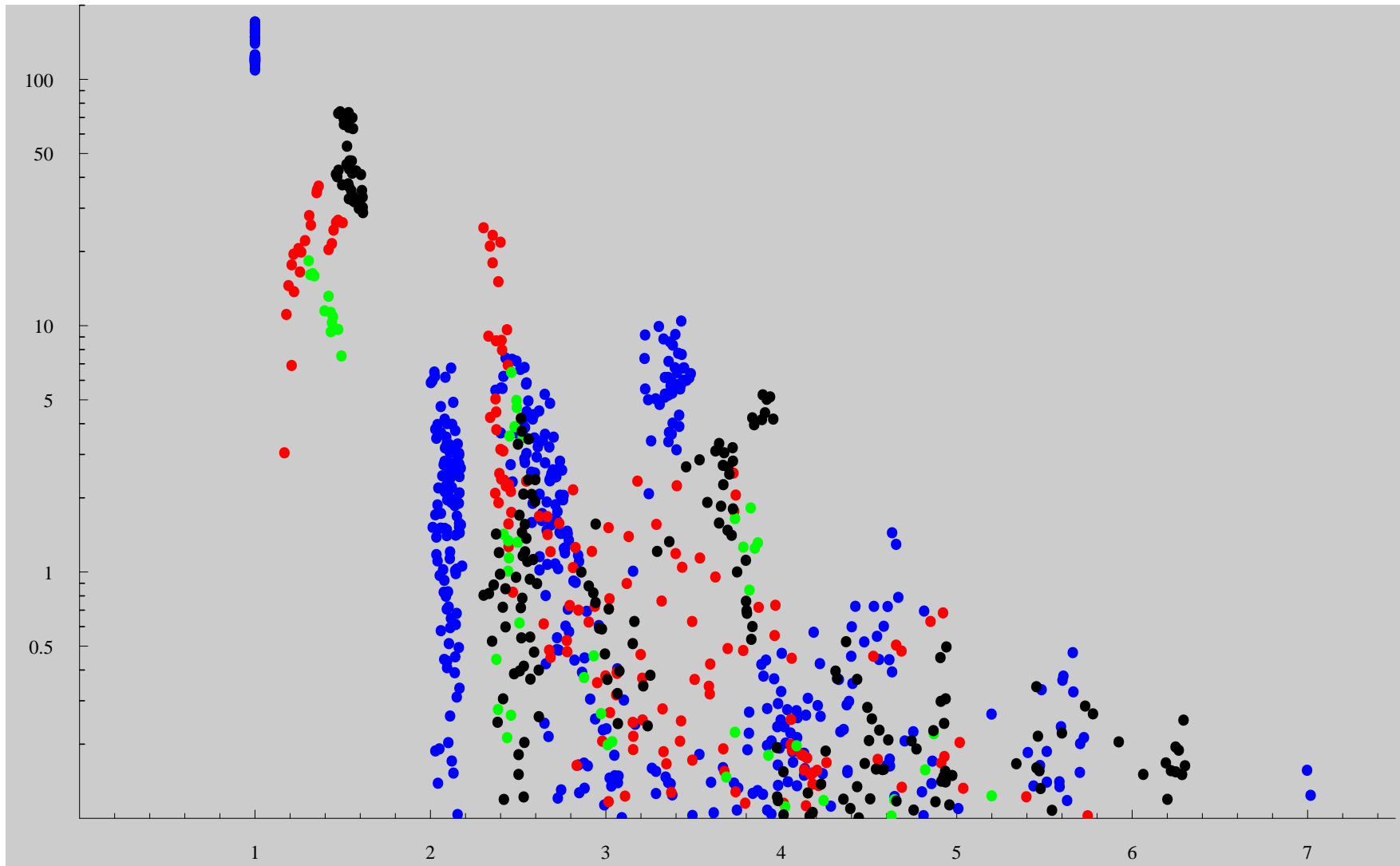


- We have calculated the FC contributions at the LDA/6-31G level of theory.



- The FC contributions fall off exponentially with increasing separation.

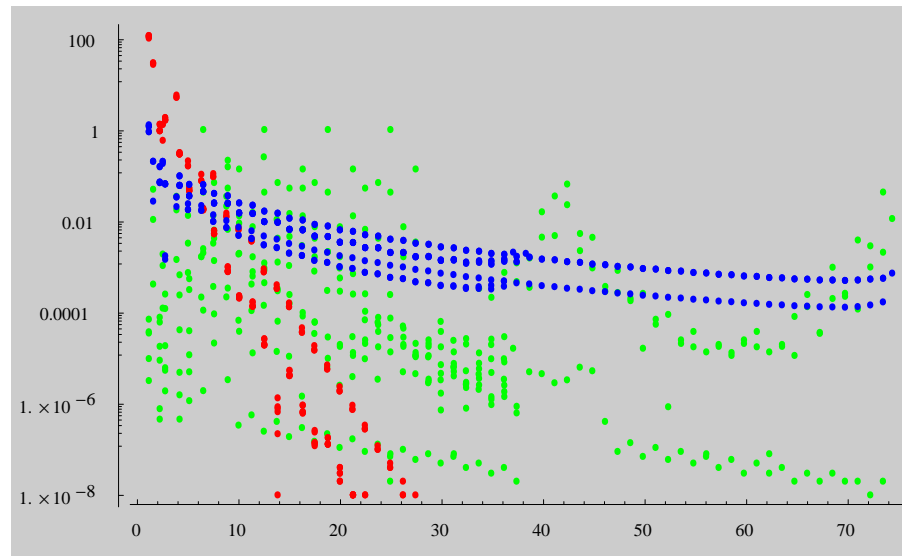
Valinomycin spin-spin couplings  $|J| > 0.1$  Hz



## Long-range behaviour of spin–spin couplings

- The FC contribution dominates at short separations but decays exponentially.
- To compare the long-distance behavior of the contributions to the spin–spin couplings, we have carried out calculations for  $C_{60}H_{120}$  at the LDA/4-31G level.
- Below we have plotted, on logarithmic scale, the contributions to the spin–spin couplings of the central and a terminal carbon atom.

- The FC contribution (red) decays exponentially.
- The DSO and PSO contributions (blue) decay as  $R^{-2}$  but tend to cancel for large separations.
- The SD contribution (green) decays in a less predictable manner.



## Conclusions

- The calculation of spin–spin coupling constants is a challenging task, requiring the solution of 9 or 10 linear equations for each magnetic nucleus.
- The calculation of indirect nuclear spin-spin couplings constants can be carried out a variety of levels of theory:
  - wave-function theory: RHF, CASSCF, RASSCF, SOPPA, CCSD
  - density-functional theory: LDA, GGA, hybrid DFT
- The most accurate results are provided by the wave-function methods, although these are mostly restricted to rather small systems.
- Because of triplet-instability problems, the RHF model is unsuitable for spin–spin calculations.
- Whereas LDA provide results of low accuracy, BLYP and in particular B3LYP provide results of almost the same quality as RASSCF, SOPPA, and CCSD.
  - current functionals have difficulties with highly electronegative elements
  - on the other hand, they work very well for hydrocarbons
- DFT can be applied to large systems, using linear-scaling techniques, opening up the possibility of studying couplings in molecules of biological interest.